Galois Theory Universitext

The book gives a detailed account of the development of the theory of algebraic equations, from its origins in ancient times to its completion by Galois in the nineteenth century. The appropriate parts of works by Cardano, Lagrange, Vandermonde, Gauss, Abel, and Galois are reviewed and placed in their historical perspective, with the aim of conveying to the reader a sense of the way in which the theory of algebraic equations has evolved and has led to such basic mathematical notions as "group" and "field". A brief discussion of the fundamental theorems of modern Galois theory and complete proofs of the quoted results are provided, and the material is organized in such a way that the more technical details can be skipped by readers who are interested primarily in a broad survey of the theory. In this second edition, the exposition has been improved throughout and the chapter on Galois has been entirely rewritten to better reflect Galois' highly innovative contributions. The text now follows more closely Galois' memoir, resorting as sparsely as possible to anachronistic modern notions such as field extensions. The emerging picture is a surprisingly elementary approach to the solvability of equations by radicals, and yet is unexpectedly close to some of the most recent methods of Galois theory.

"Artin's 1932 Göttingen Lectures on Class Field Theory" and "Connections between Algebrac Number Theory and Integral Matrices"

The exposition of the classical theory of algebraic numbers is clear and thorough, and there is a large number of exercises as well as worked out numerical examples. A careful study of this book will provide a solid background to the learning of more recent topics.

This introduction to algebraic number theory discusses the classical concepts from the viewpoint of Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

A self-contained exposition of local class field theory for students in advanced algebra.

This textbook offers a unique introduction to classical Galois theory through many concrete examples and exercises of varying difficulty (including computer-assisted exercises). In addition to covering standard material, the book explores topics related to classical problems such as Galois’ theorem on solvable groups of polynomial equations of prime degrees, Nagell's proof of non-solvability by radicals of quintic equations, Tschirnhausen's transformations, lunes of Hippocrates, and Galois' resolvents. Topics related to open conjectures are also discussed, including exercises related to the inverse Galois problem and cyclotomic fields. The author presents proofs of theorems, historical comments and useful references alongside the exercises, providing readers with a well-rounded introduction to the subject and a gateway to further reading. A valuable reference and a rich source of exercises with sample solutions, this book will be useful to both students and lecturers. Its original concept makes it particularly suitable for self-study.

This text offers a clear, efficient exposition of Galois Theory with exercises and complete proofs. Topics include: Cardano's formulas; the Fundamental Theorem; Galois' Great Theorem (solvability for radicals of a polynomial is equivalent to solvability of its Galois Group); and computation of Galois group of cubics and quartics. There are appendices on group theory and on ruler-compass constructions. Developed on the basis of a second-semester graduate algebra course, following a course on group theory, this book will provide a concise introduction to Galois Theory suitable for graduate students, either as a text for a course or for study outside the classroom.

Requiring no more than a basic knowledge of abstract algebra, this text presents the mathematics of number fields in a straightforward, pedestrian manner. It therefore avoids local methods and presents proofs in a way that highlights the important parts of the arguments. Readers are assumed to be able to fill in the details, which in many places are left as exercises.

Class field theory, which is so immediately compelling in its main assertions, has, ever since its invention, suffered from the fact that its proofs have required a complicated and, by comparison with the results, rather imper spicuous system of arguments which have tended to jump around all over the place. My earlier presentation of the theory [41] has strengthened me in the belief that a highly elaborate mechanism, such as, for example, cohomol ogy, might not be adequate for a number-theoretical law admitting a very direct formulation, and that the truth of such a law must be susceptible to a far more immediate insight. I was determined to write the present, new account of class field theory by the discovery that, in fact, both the local and the global reciprocity laws may be subsumed under a purely group theoretical principle, admitting an entirely elementary description. This de scription makes possible a new foundation for the entire theory. The rapid advance to the main theorems of class field theory which results from this approach has made it possible to include in this volume the most important consequences and elaborations, and further related theories, with the excep tion of the cohomology version which I have this time excluded. This remains a significant variant, rich in application, but its principal results should be directly obtained from the material treated here.

This is a foundation for arithmetic topology - a new branch of mathematics which is focused upon the analogy between knot theory and number theory. Starting with an informative introduction to its origins, namely Gauss, this text provides a background on knots, three manifolds and number fields. Common aspects of both knot theory and number theory, for instance knots in three manifolds versus primes in a number field, are compared throughout the book. These comparisons begin at an elementary level, slowly building up to advanced theories in later chapters. Definitions are carefully formulated and proofs are largely self-contained. When necessary, background information is provided and theory is accompanied with a number of useful examples and illustrations, making this a useful text for both undergraduates and graduates in the field of knot theory, number theory and geometry. ?

This book is designed as a text for a first-year graduate algebra course. The choice of topics is guided by the underlying theme of modules as a basic unifying concept in mathematics. Beginning with standard topics in groups and ring theory, the authors then develop basic module theory, culminating in the fundamental structure theorem for finitely generated modules over a principal ideal domain. They then treat canonical form theory in linear algebra as an application of this fundamental theorem. Module theory is also used in investigating bilinear, sesquilinear, and quadratic forms. The authors develop some multilinear algebra (Hom and tensor product) and the theory of semisimple rings and modules and apply these results in the final chapter to study group

represetations by viewing a representation of a group G over a field F as an F(G)-module. The book emphasizes proofs with a maximum of insight and a minimum of computation in order to promote understanding. However, extensive material on computation (for example, computation of canonical forms) is provided.
This is Volume II of a two-volume introductory text in classical algebra. The text moves methodically with numerous examples and details so that readers with some basic knowledge of algebra can read it without difficulty. It is recommended either as a textbook for some particular algebraic topic or as a reference book for consultations in a selected fundamental branch of algebra. The book contains a wealth of material. Amongst the topics covered in Volume are the theory of ordered fields and Nullstellen Theorems. Known researcher Lorenz also includes the fundamentals of the theory of quadratic forms, of valuations, local fields and modules. What's more, the book contains some lesser known or nontraditional results – for instance, Tsen's results on the solubility of systems of polynomial equations with a sufficiently large number of indeterminates.
In the fall of 1990, I taught Math 581 at New Mexico State University for the first time. This course on field theory is the first semester of the year-long graduate algebra course here at NMSU. In the back of my mind, I thought it would be nice someday to write a book on field theory, one of my favorite mathematical subjects, and I wrote a crude form of lecture notes that semester. Those notes sat undisturbed for three years until late in 1993 when I finally made the decision to turn the notes into a book. The notes were greatly expanded and rewritten, and they were in a form sufficient to be used as the text for Math 581 when I taught it again in the fall of 1994. Part of my desire to write a textbook was due to the nonstandard format of our graduate algebra sequence. The first semester of our sequence is field theory. Our graduate students generally pick up group and ring theory in a senior-level course prior to taking field theory. Since we start with field theory, we would have to jump into the middle of most graduate algebra textbooks. This can make reading the text difficult by not knowing what the author did before the field theory chapters. Therefore, a book devoted to field theory is desirable for us as a text. While there are a number of field theory books around, most of these were less complete than I wanted.
Galois theory is a mature mathematical subject of particular beauty. Any Galois theory book written nowadays bears a great debt to Emil Artin's classic text "Galois Theory," and this book is no exception. While Artin's book pioneered an approach to Galois theory that relies heavily on linear algebra, this book's author takes the linear algebra emphasis even further. This special approach to the subject together with the clarity of its presentation, as well as the choice of topics covered, has made the first edition of this book a more than worthwhile addition to the literature on Galois Theory. The second edition, with a new chapter on transcendental extensions, will only further serve to make the book appreciated by and approachable to undergraduate and beginning graduate math majors.

This text aims to provide graduate students with a self-contained introduction to topics that are at the forefront of modern algebra, namely, coalgebras, bialgebras and Hopf algebras. The last chapter (Chapter 4) discusses several applications of Hopf algebras, some of which are further developed in the author's 2011 publication, An Introduction to Hopf Algebras. The book may be used as the main text or as a supplementary text for a graduate algebra course. Prerequisites for this text include standard material on groups, rings, modules, algebraic extension fields, finite fields and linearly recursive sequences. The book consists of four chapters. Chapter 1 introduces algebras and coalgebras over a field K; Chapter 2 treats bialgebras; Chapter 3 discusses Hopf algebras and Chapter 4 consists of three applications of Hopf algebras. Each chapter begins with a short overview and ends with a collection of exercises which are designed to review and reinforce the material. Exercises range from straightforward applications of the theory to problems that are devised to challenge the reader. Questions for further study are provided after selected exercises. Most proofs are given in detail, though a few proofs are omitted since they are beyond the scope of this book.
This graduate textbook offers an introduction to modern methods in number theory. It gives a complete account of the main results of class field theory as well as the Poitou-Tate duality theorems, considered crowning achievements of modern number theory. Assuming a first graduate course in algebra and number theory, the book begins with an introduction to group and Galois cohomology. Local fields and local class field theory, including Lubin-Tate formal group laws, are covered next, followed by global class field theory and the description of abelian extensions of global fields. The final part of the book gives an accessible yet complete exposition of the Poitou-Tate duality theorems. Two appendices cover the necessary background in homological algebra and the analytic theory of Dirichlet L-series, including the ?ebotarev density theorem. Based on several advanced courses given by the author, this textbook has been written for graduate students. Including complete proofs and numerous exercises, the book will also appeal to more experienced mathematicians, either as a text to learn the subject or as a reference.

Lucid coverage of the major theories of abstract algebra, with helpful illustrations and exercises included throughout. Unabridged, corrected republication of the work originally published 1971. Bibliography. Index. Includes 24 tables and figures.

This is an updated English translation of Cohomologie Galoisienne, published more than thirty years ago as one of the very first versions of Lecture Notes in Mathematics. It includes a reproduction of an influential paper by R. Steinberg, together with some new material and an expanded bibliography.

Ever since the concepts of Galois groups in algebra and fundamental groups in topology emerged during the nineteenth century, mathematicians have known of the strong analogies between the two concepts. This book presents the connection starting at an elementary level, showing how the judicious use of algebraic geometry gives access to the powerful interplay between algebra and topology that underpins much modern research in geometry and number theory. Assuming as little technical background as possible, the book starts with basic algebraic and topological concepts, but already presented from the modern viewpoint advocated by Grothendieck. This enables a systematic yet accessible development of the theories of fundamental groups of algebraic curves, fundamental groups of schemes, and Tannakian fundamental groups. The connection between fundamental groups and linear differential equations is also developed at increasing levels of generality. Key applications and recent results, for example on the inverse Galois problem, are given throughout.

Galois theory is a mature mathematical subject of particular beauty. Any Galois theory book written nowadays bears a great debt to Emil Artin’s classic text "Galois Theory," and this book is no exception. While Artin’s book pioneered an approach to Galois theory that relies heavily on linear algebra, this book’s author takes the linear algebra emphasis even further. This special approach to the subject together with the clarity of its presentation, as well as the choice of topics covered, makes this book a more than worthwhile addition to the existing literature on Galois Theory. It will be appreciated by undergraduate and beginning graduate math majors.

Combining a concrete perspective with an exploration-based approach, Exploratory Galois Theory develops Galois theory at an entirely undergraduate level. The text grounds the presentation in the concept of algebraic numbers with complex approximations and assumes of its readers only a first course in abstract algebra. For readers with Maple or Mathematica, the text introduces tools for hands-on experimentation with finite extensions of the rational numbers, enabling a familiarity never before available to students of the subject. The text is appropriate for traditional lecture courses, for seminars, or for self-paced independent study by undergraduates and graduate students.

Etale cohomology is an important branch in arithmetic geometry. This book covers the main materials in SGA 1, SGA 4, SGA 4 1/2 and SGA 5 on etale cohomology theory, which includes decent theory, etale fundamental groups, Galois cohomology, etale cohomology, derived categories, base change theorems, duality, and ?-adic cohomology. The prerequisites for reading this book are basic algebraic geometry and advanced commutative algebra.

The problems are systematically arranged to reveal the evolution of concepts and ideas of the subject Includes various levels of problems - some are easy and straightforward, while others are more challenging All problems are elegantly solved

This book stems from lectures on commutative algebra for 4th-year university students at two French universities (Paris and Rennes). At that level, students have already followed a basic course in linear algebra and are essentially fluent with the language of vector spaces over fields. The topics introduced include arithmetic of rings, modules, especially principal ideal rings and the classification of modules over such rings, Galois theory, as well as an introduction to more advanced topics such as homological algebra, tensor products, and algebraic concepts involved in algebraic geometry. More than 300 exercises will allow the reader to deepen his understanding of the subject. The book also includes 11 historical vignettes about mathematicians who contributed to commutative algebra.

Algebraic number theory introduces students not only to new algebraic notions but also to related concepts: groups, rings, fields, ideals, quotient rings and quotient fields, homomorphisms and isomorphisms, modules, and vector spaces. Author Pierre

Samuel notes that students benefit from their studies of algebraic number theory by encountering many concepts fundamental to other branches of mathematics — algebraic geometry, in particular. This book assumes a knowledge of basic algebra but supplements its teachings with brief, clear explanations of integrality, algebraic extensions of fields, Galois theory, Noetherian rings and modules, and rings of fractions. It covers the basics, starting with the divisibility theory in principal ideal domains and ending with the unit theorem, finiteness of the class number, and the more elementary theorems of Hilbert ramification theory. Numerous examples, applications, and exercises appear throughout the text.

Class field theory brings together the quadratic and higher reciprocity laws of Gauss, Legendre, and others, and vastly generalizes them. This book provides an accessible introduction to class field theory. It takes a traditional approach in that it attempts to present the material using the original techniques of proof, but in a fashion which is cleaner and more streamlined than most other books on this topic. It could be used for a graduate course on algebraic number theory, as well as for students who are interested in self-study. The book has been class-tested, and the author has included lots of challenging exercises throughout the text.

Providing a broad review of many techniques and their application to condensed matter systems, this book begins with a review of thermodynamics and statistical mechanics, before moving onto real and imaginary time path integrals and the link between Euclidean quantum mechanics and statistical mechanics. A detailed study of the Ising, gauge-Ising and XY models is included. The renormalization group is developed and applied to critical phenomena, Fermi liquid theory and the renormalization of field theories. Next, the book explores bosonization and its applications to one-dimensional fermionic systems and the correlation functions of homogeneous and random-bond Ising models. It concludes with Bohm–Pines and Chern–Simons theories applied to the quantum Hall effect. Introducing the reader to a variety of techniques, it opens up vast areas of condensed matter theory for both graduate students and researchers in theoretical, statistical and condensed matter physics.

Galois TheorySpringer Science & Business Media

The Standard Model is the foundation of modern particle and high energy physics. This book explains the mathematical background behind the Standard Model, translating ideas from physics into a mathematical language and vice versa. The first part of the book covers the mathematical theory of Lie groups and Lie algebras, fibre bundles, connections, curvature and spinors. The second part then gives a detailed exposition of how these concepts are applied in physics, concerning topics such as the Lagrangians of gauge and matter fields, spontaneous symmetry breaking, the Higgs boson and mass generation of gauge bosons and fermions. The book also contains a chapter on advanced and modern topics in particle physics, such as neutrino masses, CP violation and Grand Unification. This carefully written textbook is aimed at graduate students of mathematics and physics. It contains numerous examples and more than 150 exercises, making it suitable for self-study and use alongside lecture courses. Only a basic knowledge of differentiable manifolds and special relativity is required, summarized in the appendix.

This book links two subjects: algebraic geometry and coding theory. It uses a novel approach based on the theory of algebraic function fields. Coverage includes the

Riemann-Rock theorem, zeta functions and Hasse-Weil's theorem as well as Goppa' s algebraic-geometric codes and other traditional codes. It will be useful to researchers in algebraic geometry and coding theory and computer scientists and engineers in information transmission.

The landscape of homological algebra has evolved over the last half-century into a fundamental tool for the working mathematician. This book provides a unified account of homological algebra as it exists today. The historical connection with topology, regular local rings, and semi-simple Lie algebras are also described. This book is suitable for second or third year graduate students. The first half of the book takes as its subject the canonical topics in homological algebra: derived functors, Tor and Ext, projective dimensions and spectral sequences. Homology of group and Lie algebras illustrate these topics. Intermingled are less canonical topics, such as the derived inverse limit functor lim1, local cohomology, Galois cohomology, and affine Lie algebras. The last part of the book covers less traditional topics that are a vital part of the modern homological toolkit: simplicial methods, Hochschild and cyclic homology, derived categories and total derived functors. By making these tools more accessible, the book helps to break down the technological barrier between experts and casual users of homological algebra.

A clear, efficient exposition of this topic with complete proofs and exercises, covering cubic and quartic formulas; fundamental theory of Galois theory; insolvability of the quintic; Galoiss Great Theorem; and computation of Galois groups of cubics and quartics. Suitable for first-year graduate students, either as a text for a course or for study outside the classroom, this new edition has been completely rewritten in an attempt to make proofs clearer by providing more details. It now begins with a short section on symmetry groups of polygons in the plane, for there is an analogy between polygons and their symmetry groups and polynomials and their Galois groups - an analogy which serves to help readers organise the various field theoretic definitions and constructions. The text is rounded off by appendices on group theory, ruler-compass constructions, and the early history of Galois Theory. The exposition has been redesigned so that the discussion of solvability by radicals now appears later and several new theorems not found in the first edition are included.

Eugène Charles Catalan made his famous conjecture – that 8 and 9 are the only two consecutive perfect powers of natural numbers – in 1844 in a letter to the editor of Crelle’s mathematical journal. One hundred and fifty-eight years later, Preda Mihailescu proved it. Catalan’s Conjecture presents this spectacular result in a way that is accessible to the advanced undergraduate. The author dissects both Mihailescu’s proof and the earlier work it made use of, taking great care to select streamlined and transparent versions of the arguments and to keep the text self-contained. Only in the proof of Thaine’s theorem is a little class field theory used; it is hoped that this application will motivate the interested reader to study the theory further. Beautifully clear and concise, this book will appeal not only to specialists in number theory but to anyone interested in seeing the application of the ideas of algebraic number theory to a famous mathematical problem.

This translation of the 1987 German edition is an introduction into the classical

parts of algebra with a focus on fields and Galois theory. It discusses nonstandard topics, such as the transcendence of pi, and new concepts are defined in the framework of the development of carefully selected problems. It includes an appendix with exercises and notes on the previous parts of the book, and brief historical comments are scattered throughout.
Easily accessible Includes recent developments Assumes very little knowledge of differentiable manifolds and functional analysis Particular emphasis on topics related to mirror symmetry (SUSY, Kaehler-Einstein metrics, Tian-Todorov lemma)
The constructive approach to mathematics has enjoyed a renaissance, caused in large part by the appearance of Errett Bishop's book Foundations of constr"uctiue analysis in 1967, and by the subtle influences of the proliferation of powerful computers. Bishop demonstrated that pure mathematics can be developed from a constructive point of view while maintaining a continuity with classical terminology and spirit; much more of classical mathematics was preserved than had been thought possible, and no classically false theorems resulted, as had been the case in other constructive schools such as intuitionism and Russian constructivism. The computers created a widespread awareness of the intuitive notion of an effecti ve procedure, and of computation in principle, in addi tion to stimulating the study of constructive algebra for actual implementation, and from the point of view of recursive function theory. In analysis, constructive problems arise instantly because we must start with the real numbers, and there is no finite procedure for deciding whether two given real numbers are equal or not (the real numbers are not discrete) . The main thrust of constructive mathematics was in the direction of analysis, although several mathematicians, including Kronecker and van der waerden, made important contributions to construc tive algebra. Heyting, working in intuitionistic algebra, concentrated on issues raised by considering algebraic structures over the real numbers, and so developed a handmaiden'of analysis rather than a theory of discrete algebraic structures.
All needed notions are developed within the book: with the exception of fundamentals which are presented in introductory lectures, no other knowledge is assumed Provides a more in-depth introduction to the subject than other existing books in this area Over 400 exercises including hints for solutions are included
Modern number theory began with the work of Euler and Gauss to understand and extend the many unsolved questions left behind by Fermat. In the course of their investigations, they uncovered new phenomena in need of explanation, which over time led to the discovery of field theory and its intimate connection with complex multiplication. While most texts concentrate on only the elementary or advanced aspects of this story, Primes of the Form x2 + ny2 begins with Fermat and explains how his work ultimately gave birth to quadratic reciprocity and the genus theory of quadratic forms. Further, the book shows how the results of Euler and Gauss can be fully understood only in the context of class field theory. Finally, in order to bring class field theory down to earth, the book

explores some of the magnificent formulas of complex multiplication. The central theme of the book is the story of which primes p can be expressed in the form x2 + ny2. An incomplete answer is given using quadratic forms. A better though abstract answer comes from class field theory, and finally, a concrete answer is provided by complex multiplication. Along the way, the reader is introduced to some wonderful number theory. Numerous exercises and examples are included. The book is written to be enjoyed by readers with modest mathematical backgrounds. Chapter 1 uses basic number theory and abstract algebra, while chapters 2 and 3 require Galois theory and complex analysis, respectively.