Topics In Number Theory Algebra And Geometry

DIVBasic treatment, incorporating language of abstract algebra and a history of the discipline. Unique factorization and the GCD, quadratic residues, sums of squares, much more. Numerous problems. Bibliography. 1977 edition. /div

Lectures on Number Theory is the first of its kind on the subject matter. It covers most of the topics that are standard in a modern first course on number theory, but also includes Dirichlet's famous results on class numbers and primes in arithmetic progressions.

This volume honors Sir Peter Swinnerton-Dyer's mathematical career spanning more than 60 years' of amazing creativity in number theory and algebraic geometry.

The book attempts to point out the interconnections between number theory and algebra with a view to making a student understand certain basic concepts in the two areas forming the subject-matter of the book.

The book is primarily intended as a textbook on modern algebra for undergraduate mathematics students. It is also useful for those who are interested in supplementary reading at a higher level. The text is designed in such a way that it encourages independent thinking and motivates students towards further study. The book covers all major topics in group, ring, vector space and module theory that are usually contained in a standard modern algebra text. In addition, it studies semigroup, group action, Hopf's group, topological groups and Lie groups with their actions, applications of ring theory to algebraic geometry, and defines Zariski topology, as well as applications of module theory to structure theory of rings and homological algebra. Algebraic aspects of classical number theory and algebraic number theory are also discussed with an eye to developing modern cryptography. Topics on applications to algebraic topology, category theory, algebraic geometry, algebraic number theory, cryptography and theoretical computer science interlink the subject with different areas. Each chapter discusses individual topics, starting from the basics, with the help of illustrative examples. This comprehensive text with a broad variety of concepts, applications, examples, exercises and historical notes represents a valuable and unique resource.

One of the oldest branches of mathematics, number theory is a vast field devoted to studying the properties of whole numbers. Offering a flexible format for a one- or two-semester course, Introduction to Number Theory uses worked examples, numerous exercises, and two popular software packages to describe a diverse array of number theory topics. This classroom-tested, student-friendly text covers a wide range of subjects, from the ancient Euclidean algorithm for finding the greatest common divisor of two integers to recent developments that include cryptography, the theory of elliptic curves, and the negative solution of Hilbert’s tenth problem. The authors illustrate the connections between number theory and other areas of mathematics, including algebra, analysis, and combinatorics. They also describe applications of number theory to real-world problems, such as congruences in the ISBN system, modular arithmetic and Euler’s theorem in RSA encryption, and quadratic residues in the construction of tournaments. The book interweaves the theoretical development of the material with Mathematica® and MapleTM calculations while giving brief tutorials on the software in the appendices. Highlighting both fundamental and advanced topics, this introduction provides all of the tools to achieve a solid foundation in number theory.

This textbook covers the main topics in number theory as taught in universities throughout the world. Number theory deals mainly with properties of integers and rational numbers; it is not an organized theory in the usual sense but a vast collection of individual topics and results, with some coherent sub-theories and a long list of unsolved problems. This book excludes topics relying heavily on complex analysis and advanced algebraic number theory. The increased use of computers in number theory is reflected in many sections (with much greater emphasis in this edition). Some results of a more advanced nature are also given, including the Gelfond-Schneider theorem, the prime number theorem, and the Mordell-Weil theorem. The latest work on Fermat's last theorem is also briefly discussed. Each chapter ends with a collection of problems; hints or sketch solutions are given at the end of the book, together with various useful tables.

Challenging, accessible mathematical adventures involving prime numbers, number patterns, irrationals and iterations, calculating prodigies, and more. No special training is needed, just high school mathematics and an inquisitive mind. "A splendidly written, well selected and presented collection. I recommend the book unreservedly to all readers." — Martin Gardner.

Number theory is one of the largest and most popular subject areas in mathematics, and this book is a superb entry to the subject. It features a well-known international author and covers enough material to satisfy both students and the serious researcher. A splendid addition to the marque series of the AMS publishing program.

The title of this book may be read in two ways. One is 'algebraic number-theory', that is, the theory of numbers viewed algebraically; the other, 'algebraic-number theory', the study of algebraic numbers. Both readings are compatible with our aims, and both are perhaps misleading. Misleading, because a proper coverage of either topic would require more space than is available, and demand more of the reader than we wish to; compatible, because our aim is to illustrate how some of the basic notions of the theory of algebraic numbers may be applied to problems in number theory. Algebra is an easy subject to compartmentalize, with topics such as 'groups', 'rings' or 'modules' being taught in comparative isolation. Many students view it this way. While it would be easy to exaggerate this tendency, it is not an especially desirable one. The leading mathematicians of the nineteenth and early twentieth centuries developed and used most of the basic results and techniques of linear algebra for perhaps a hundred years, without ever defining an abstract vector space: nor is there anything to suggest that they suf fered thereby. This historical fact may indicate that abstrac tion is not always as necessary as one commonly imagines; on the other hand the axiomatization of mathematics has led to enormous organizational and conceptual gains.

The fields of algebraic functions of one variable appear in several areas of mathematics: complex analysis, algebraic geometry, and number theory. This text adopts the latter perspective by applying an arithmetic-algebraic viewpoint to the study of function fields as part of the algebraic theory of numbers. The examination explains both the similarities and fundamental differences between function fields and number fields, including many exercises and examples to enhance understanding and motivate further study. The only prerequisites are a basic knowledge of field theory, complex analysis, and some commutative algebra.

At the time of Professor Rademacher's death early in 1969, there was available a complete manuscript of the present work. The editors had only to supply a few bibliographical references and to correct a few misprints and errors. No substantive changes were made in the manu script except in one or two places where references to additional material appeared; since this material was not found in Rademacher's papers, these references were deleted. The editors are grateful to Springer-Verlag for their helpfulness and courtesy. Rademacher started work on the present volume no later than 1944; he was still working on it at the inception of his final illness. It represents the parts of analytic number theory that were of greatest interest to him. The editors, his students, offer this work as homage to the memory of a great man to whom they, in common with all number theorists, owe a deep and lasting debt. E. Grosswald Temple University, Philadelphia, PA 19122, U.S.A. J. Lehner University of Pittsburgh, Pittsburgh, PA 15213 and National Bureau of Standards, Washington, DC 20234, U.S.A. M. Newman National Bureau of Standards, Washington, DC 20234, U.S.A. Contents I. Analytic tools Chapter 1. Bernoulli polynomials and Bernoulli numbers .... 1 1. The binomial coefficients .... 1 2. The Bernoulli polynomials .... 4 3. Zeros of the Bernoulli polynomials .... 7 4. The Bernoulli numbers .... 9 5. The von Staudt-Clausen theorem .... 10 6. A multiplication formula for the Bernoulli polynomials ....

Now in its second edition, this textbook provides an introduction and overview of number theory based on the density and properties of the prime numbers. This unique approach offers both a firm background in the standard material of number theory, as well as an overview of the entire discipline. All of the essential topics are covered, such as the fundamental theorem of arithmetic, theory of congruences, quadratic reciprocity, arithmetic functions, and the distribution of primes. New in this edition are coverage of p-adic numbers, Hensel's lemma, multiple zeta-values, and elliptic curve methods in primality testing. Key topics and features include: A solid introduction to analytic number theory, including full proofs of Dirichlet's Theorem and the Prime Number Theorem Concise treatment of algebraic number theory, including a complete presentation of primes, prime factorizations in algebraic number fields, and unique factorization of ideals Discussion of the AKS algorithm, which shows that primality testing is one of polynomial time, a topic not usually included in such texts Many interesting ancillary topics, such as primality testing and cryptography, Fermat and Mersenne numbers, and Carmichael numbers The user-friendly style, historical context, and wide range of exercises that range from simple to quite difficult (with solutions and hints provided for select exercises) make Number Theory: An Introduction via the Density of Primes ideal for both self-study and classroom use. Intended for upper level undergraduates and beginning graduates, the only prerequisites are a basic knowledge of calculus, multivariable calculus, and some linear algebra. All necessary concepts from abstract algebra and complex analysis are introduced where needed.

Classic 2-part work now available in a single volume. Contents range from chapters on binary quadratic forms to the Thue-Siegel-Roth Theorem and the Prime Number Theorem. Includes problems and solutions. 1956 edition.

Number Theory Revealed: A Masterclass acquaints enthusiastic students with the “Queen of Mathematics”. The text offers a fresh take on congruences, power residues, quadratic residues, primes, and Diophantine equations and presents hot topics like cryptography, factoring, and primality testing. Students are also introduced to beautiful enlightening questions like the structure of Pascal's triangle mod p and modern twists on traditional questions like the values represented by binary quadratic forms, the anatomy of integers, and elliptic curves. This Masterclass edition contains many additional chapters and appendices not found in Number Theory Revealed: An Introduction, highlighting beautiful developments and inspiring other subjects in mathematics (like algebra). This allows instructors to tailor a course suited to their own (and their students') interests. There are new yet accessible topics like the curvature of circles in a tiling of a circle by circles, the latest discoveries on gaps between primes, a new proof of Mordell's Theorem for congruent elliptic curves, and a discussion of the abc-conjecture including its proof for polynomials. About the Author: Andrew Granville is the Canada Research Chair in Number Theory at the University of Montreal and professor of mathematics at University College London. He has won several international writing prizes for exposition in mathematics, including the 2008 Chauvenet Prize and the 2019 Halmos-Ford Prize, and is the author of Prime Suspects (Princeton University Press, 2019), a beautifully illustrated graphic novel murder mystery that explores surprising connections between the anatomies of integers and of permutations.

An Introduction to Commutative Algebra and Number Theory is an elementary introduction to these subjects. Beginning with a concise review of groups, rings and fields, the author presents topics in algebra from a distinctly number-theoretic perspective and sprinkles number theory results throughout his presentation. The topics in algebra include polynomial rings, UFD, PID, and Euclidean domains; and field extensions, modules, and Dedekind domains. In the section on number theory, in addition to covering elementary congruence results, the laws of quadratic reciprocity and basics of algebraic number fields, this book gives glimpses into some deeper aspects of the subject. These include Warning's and Chevally's theorems in the finite field sections, and many results of additive number theory, such as the derivation of LaGrange's four-square theorem from Minkowski's result in the geometry of numbers. With addition of remarks and comments and with references in the bibliography, the author stimulates readers to explore the subject beyond the scope of this book.

Carl Friedrich Gauss’s textbook, Disquisitiones arithmeticae, published in 1801 (Latin), remains to this day a true masterpiece of mathematical examination. .

The aim of this book is to familiarize the reader with fundamental topics in number theory: theory of divisibility, arithmetrical functions, prime numbers, geometry of numbers, additive number theory, probabilistic number theory, theory of Diophantine approximations and algebraic number theory. The author tries to show the connection between number theory and other branches of mathematics with the resultant tools adopted in the book ranging from algebra to probability theory, but without exceeding the undergraduate students who wish to be acquainted with number theory, graduate students intending to specialize in this field and researchers requiring the present state of knowledge.

These notes serve as course notes for an undergraduate course in number theory. Most if not all universities worldwide offer introductory courses in number theory for math majors and in many cases as an elective course. The notes contain a useful introduction to important topics that need to be addressed in a course in number theory. Proofs of basic theorems are presented in an interesting and comprehensive way that can be read and understood even by non-majors with the exception in the last three chapters where a background in analysis, measure theory and abstract algebra is required. The exercises are carefully chosen to broaden the understanding of the concepts. Moreover, these notes shed light on analytic number theory, a subject that is rarely seen or approached by undergraduate students. One of the unique characteristics of these notes is the careful choice of topics and its importance in the theory of numbers. The freedom is given in the last two chapters because of the advanced nature of the topics that are presented.

Number Theory is more than a comprehensive treatment of the subject. It is an introduction to topics in higher level mathematics, and unique in its scope; topics from analysis, modern algebra, and discrete mathematics are all included.

The book is divided into two parts. Part A covers key concepts of number theory and could serve as a first course on the subject. Part B delves into more advanced topics and an exploration of related mathematics. The prerequisites for this self-contained text are elements from linear algebra. Valuable references for the reader are collected at the end of each chapter. It is suitable as an introduction to higher level mathematics for undergraduates, or for self-study.
This introductory book emphasises algorithms and applications, such as cryptography and error correcting codes.
Elements of the Theory of Numbers teaches students how to develop, implement, and test numerical methods for standard mathematical problems. The authors have created a two-pronged pedagogical approach that integrates analysis and algebra with classical number theory. Making greater use of the language and concepts in algebra and analysis than is traditionally encountered in introductory courses, this pedagogical approach helps to instill in the minds of the students the idea of the unity of mathematics. Elements of the Theory of Numbers is a superb summary of classical material as well as allowing the reader to take a look at the exciting role of analysis and algebra in number theory. * In-depth coverage of classical number theory * Thorough discussion of the theory of groups and rings * Includes application of Taylor polynomials * Contains more advanced material than other texts * Illustrates the results of a theorem with an example * Excellent presentation of the standard computational exercises * Nearly 1000 problems--many are proof-oriented, several others require the writing of computer programs to complete the computations * Clear and well-motivated presentation * Provides historical references noting distinguished number theory luminaries such as Euclid, de Fermat, Hilbert, Brun, and Lehmer, to name a few * Annotated bibliographies appear at the end of all of the chapters
This book offers multiple interconnected perspectives on the largely untapped potential of elementary number theory for mathematics education: its formal and cognitive nature, its relation to arithmetic and algebra, its accessibility, its utility and intrinsic merits, to name just a few. Its purpose is to promote explication and critical dialogue about these issues within the international mathematics education community. The studies comprise a variety of pedagogical and research orientations by an international group of researchers that, collectively, make a compelling case for the relevance and importance of number theory in mathematics education in both pre K-16 settings and mathematics teacher education. Topics variously engaged include: *understanding particular concepts related to numerical structure and number theory; *elaborating on the historical and psychological relevance of number theory in concept development; *attaining a smooth transition and extension from pattern recognition to formative principles; *appreciating the aesthetics of number structure; *exploring its suitability in terms of making connections leading to aha! insights and reaching toward the learner's affective domain; *reexamining previously constructed knowledge from a novel angle; *investigating connections between technique and theory; *utilizing computers and calculators as pedagogical tools; and *generally illuminating the role

number theory concepts could play in developing mathematical knowledge and reasoning in students and teachers. Overall, the chapters of this book highlight number theory-related topics as a stepping-stone from arithmetic toward generalization and algebraic formalism, and as a means for providing intuitively grounded meanings of numbers, variables, functions, and proofs. Number Theory in Mathematics Education: Perspectives and Prospects is of interest to researchers, teacher educators, and students in the field of mathematics education, and is well suited as a text for upper-level mathematics education courses.

A look at solving problems in three areas of classical elementary mathematics: equations and systems of equations of various kinds, algebraic inequalities, and elementary number theory, in particular divisibility and diophantine equations. In each topic, brief theoretical discussions are followed by carefully worked out examples of increasing difficulty, and by exercises which range from routine to rather more challenging problems. While it emphasizes some methods that are not usually covered in beginning university courses, the book nevertheless teaches techniques and skills which are useful beyond the specific topics covered here. With approximately 330 examples and 760 exercises.

The problems are systematically arranged to reveal the evolution of concepts and ideas of the subject Includes various levels of problems - some are easy and straightforward, while others are more challenging All problems are elegantly solved

This is a second edition of Lang's well-known textbook. It covers all of the basic material of classical algebraic number theory, giving the student the background necessary for the study of further topics in algebraic number theory, such as cyclotomic fields, or modular forms. "Lang's books are always of great value for the graduate student and the research mathematician. This updated edition of Algebraic number theory is no exception."—-MATHEMATICAL REVIEWS

This book presents state-of-the-art research and survey articles that highlight work done within the Priority Program SPP 1489 “Algorithmic and Experimental Methods in Algebra, Geometry and Number Theory”, which was established and generously supported by the German Research Foundation (DFG) from 2010 to 2016. The goal of the program was to substantially advance algorithmic and experimental methods in the aforementioned disciplines, to combine the different methods where necessary, and to apply them to central questions in theory and practice. Of particular concern was the further development of freely available open source computer algebra systems and their interaction in order to create powerful new computational tools that transcend the boundaries of the individual disciplines involved. The book covers a broad range of topics addressing the design and theoretical foundations, implementation and the successful application of algebraic algorithms in order to solve mathematical research problems. It offers a valuable resource for all researchers, from graduate students through established experts, who are interested in the computational aspects of algebra, geometry, and/or number theory.

This undergraduate textbook provides an approachable and thorough introduction to the topic of algebraic number theory, taking the reader from unique factorisation in the integers through to the modern-day number field sieve. The first few chapters consider the importance of arithmetic in fields larger than the rational numbers. Whilst some results generalise well, the unique factorisation of the integers in these more general number fields often fail. Algebraic number theory aims to overcome this problem. Most examples are taken from quadratic fields, for which calculations are easy to perform. The middle section considers more general theory and results for number fields, and the book concludes with some topics which are more likely to be suitable for advanced students, namely, the analytic class number formula and the number field sieve. This is the first time that the number field sieve has been considered in a textbook at this level.

* Contains a selection of articles exploring geometric approaches to problems in algebra, algebraic geometry and number theory * The collection gives a representative sample of problems and most recent results in algebraic and arithmetic geometry * Text can serve as an intense introduction for graduate students and those wishing to pursue research in algebraic and arithmetic geometry

From July 31 through August 3,1997, the Pennsylvania State University hosted the Topics in Number Theory Conference. The conference was organized by Ken Ono and myself. By writing the preface, I am afforded the opportunity to express my gratitude to Ken for beng the inspiring and driving force behind the whole conference. Without his energy, enthusiasm and skill the entire event would never have occurred. We are extremely grateful to the sponsors of the conference: The National Sci ence Foundation, The Penn State Conference Center and the Penn State Depart ment of Mathematics. The object in this conference was to provide a variety of presentations giving a current picture of recent, significant work in number theory. There were eight plenary lectures: H. Darmon (McGill University), "Non-vanishing of L-functions and their derivatives modulo p. " A. Granville (University of Georgia), "Mean values of multiplicative functions. " C. Pomerance (University of Georgia), "Recent results in primality testing. " C. Skinner (Princeton University), "Deformations of Galois representations. " R. Stanley (Massachusetts Institute of Technology), "Some interesting hyperplane arrangements. " F. Rodriguez Villegas (Princeton University), "Modular Mahler measures. " T. Wooley (University of Michigan), "Diophantine problems in many variables: The role of additive number theory. " D. Zeilberger (Temple University), "Reverse engineering in combinatorics and number theory. " The papers in this volume provide an accurate picture of many of the topics presented at the conference including contributions from four of the plenary lectures.

Ideal either for classroom use or as exercises for mathematically minded individuals, this text introduces elementary valuation theory, extension of valuations, local and ordinary arithmetic fields, and global, quadratic, and cyclotomic fields. Solutions of equations in integers is the central problem of number theory and is the focus of this book. The amount of material is suitable for a one-semester course. The author has tried to avoid the ad hoc proofs in favor of unifying ideas that work in many situations. There are exercises at the end of almost every section, so that each new idea or proof receives immediate reinforcement.

Undergraduate text uses combinatorial approach to accommodate both math majors and liberal arts students. Covers the basics of number theory, offers an outstanding introduction to partitions, plus chapters on multiplicativity-divisibility, quadratic congruences, additivity, and more

Explore the main algebraic structures and number systems that play a central role across the field of mathematics Algebra and number theory are two powerful branches of modern mathematics at the forefront of current mathematical research, and each plays an increasingly significant role in different branches of mathematics, from geometry and topology to computing and communications. Based on the authors' extensive experience within the field, Algebra and Number Theory has an innovative approach that integrates three disciplines—linear algebra, abstract algebra, and number theory—into one comprehensive and fluid presentation, facilitating a deeper understanding of the topic and improving readers' retention of the main concepts. The book begins with an introduction to the elements of set theory. Next, the authors discuss matrices, determinants, and elements of field theory, including preliminary information related to integers and complex numbers. Subsequent chapters explore key ideas relating to linear algebra such as vector spaces, linear mapping, and bilinear forms. The book explores the development of the main ideas of algebraic structures and concludes with applications of algebraic ideas to number theory. Interesting applications are provided throughout to demonstrate the relevance of the discussed concepts. In addition, chapter exercises allow readers to test their comprehension of the presented material. Algebra and Number Theory is an excellent book for courses on linear algebra, abstract algebra, and number theory at the upper-undergraduate level. It is also a valuable reference for researchers working in different fields of mathematics, computer science, and engineering as well as for individuals preparing for a career in mathematics education.

Many of the important and creative developments in modern mathematics resulted from attempts to solve questions that originate in number theory. The publication of Emil Grosswald’s classic text presents an illuminating introduction to number theory. Combining the historical developments with the analytical approach, Topics from the Theory of Numbers offers the reader a diverse range of subjects to investigate.

Algebraic number theory introduces students not only to new algebraic notions but also to related concepts: groups, rings, fields, ideals, quotient rings and quotient fields, homomorphisms and isomorphisms, modules, and vector spaces. Author Pierre Samuel notes that students benefit from their studies of algebraic number theory by encountering many concepts fundamental to other branches of mathematics — algebraic geometry, in particular. This book assumes a knowledge of basic algebra but supplements its teachings with brief, clear explanations of integrality, algebraic extensions of fields, Galois theory, Noetherian rings and modules, and rings of fractions. It covers the basics, starting with the divisibility theory in principal ideal domains and ending with the unit theorem, finiteness of the class number, and the more elementary theorems of Hilbert ramification theory. Numerous examples, applications, and exercises appear throughout the text.

This book provides an introduction and overview of number theory based on the distribution and properties of primes. This unique approach provides both a firm background in the standard material as well as an overview of the whole discipline. All the essential topics are covered: fundamental theorem of arithmetic, theory of congruences, quadratic reciprocity, arithmetic functions, and the distribution of primes. Analytic number theory and algebraic number theory both receive a solid introductory treatment. The book’s user-friendly style, historical context, and wide range of exercises make it ideal for self study and classroom use.

In recent years, number theory and arithmetic geometry have been enriched by new techniques from noncommutative geometry, operator algebras, dynamical systems, and K-Theory. This volume collects and presents up-to-date research topics in arithmetic and noncommutative geometry and ideas from physics that point to possible new connections between the fields of number theory, algebraic geometry and noncommutative geometry. The articles collected in this volume present new noncommutative geometry perspectives on classical topics of number theory and arithmetic such as modular forms, class field theory, the theory of reductive p-adic groups, Shimura varieties, the local L-factors of arithmetic varieties. They also show how arithmetic appears naturally in noncommutative geometry and in physics, in the residues of Feynman graphs, in the properties of noncommutative tori, and in the quantum Hall effect.